CHAPTER 8

Sinusoidal Steady State Analysis

8.1. General Approach

In the previous chapter, we have learned that the steady-state response of a circuit to sinusoidal inputs can be obtained by using phasors. In this chapter, we present many examples in which nodal analysis, mesh analysis, Thevenin's theorem, superposition, and source transformations are applied in analyzing ac circuits.

8.1.1. **Steps to analyze ac circuits, using phasor domain:**

phasor domain

Step 1. Transform the circuit to the phasor or frequency domain.

- Not necessary if the problem is specified in the frequency domain.

Step 2. Solve the problem using circuit techniques (e.g., nodal analysis, mesh analysis, Thevenin's theorem, superposition, or source transformations)

- The analysis is performed in the same manner as dc circuit analysis except that complex numbers are involved.

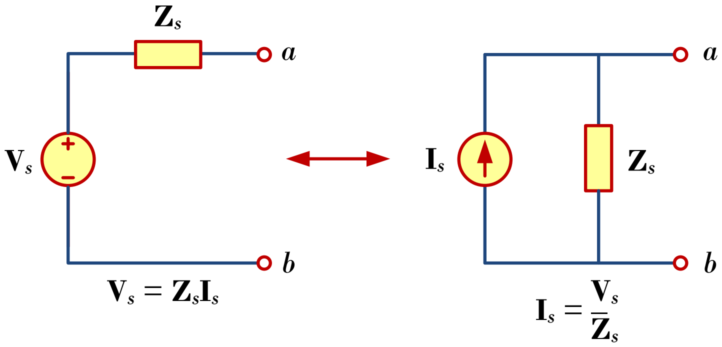
Step 3. Transform the resulting phasor back to the time domain.

8.1.2. ac circuits are linear (they are just composed of sources and impedances)

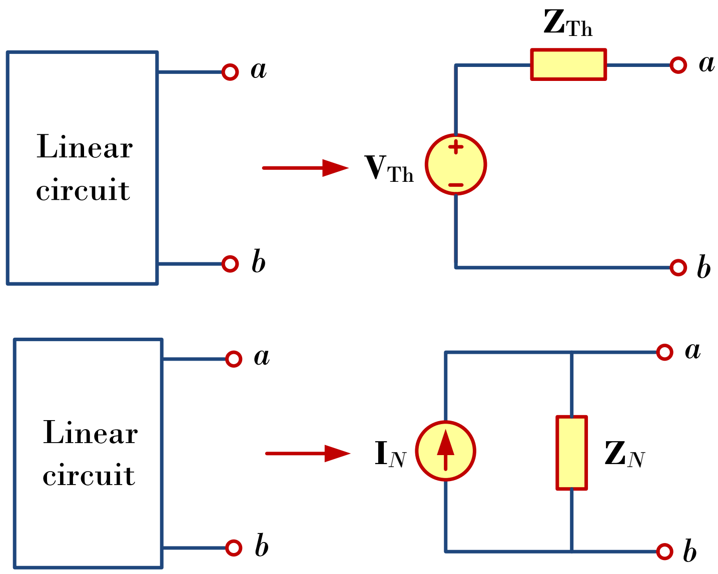
8.1.3. The **superposition theorem** applies to ac circuits the same way it applies to dc circuits. This is the case when all the sources in the circuit operate at the same frequency. If they are operating at different frequency, see Section 8.2.

8.1.4. **Source transformation**:

$$\mathbf{V_s} = \mathbf{Z_s}\mathbf{I_s}, \quad \mathbf{I_s} = \frac{\mathbf{V_s}}{\mathbf{Z_s}}.$$



8.1.5. **Thevenin and Norton Equivalent circuits**:

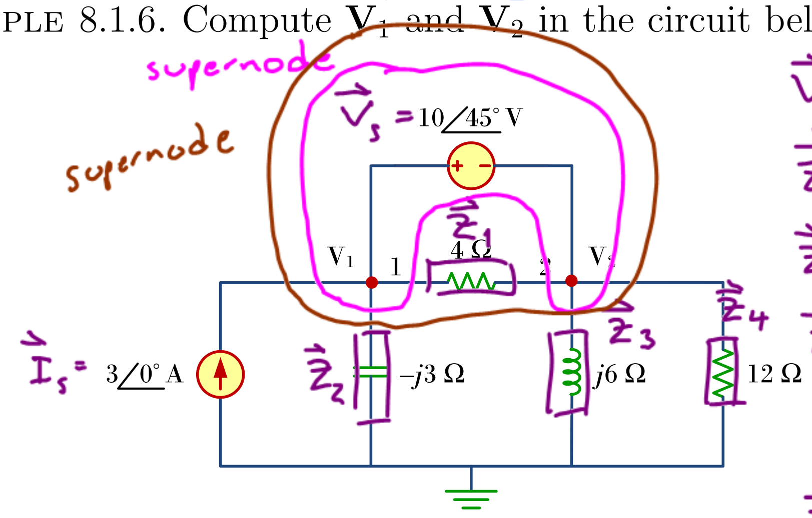


$$\mathbf{V_{Th}} = \mathbf{Z_N}\mathbf{I_N}, \quad \mathbf{Z_{Th}} = \mathbf{Z_N}$$

V_1 $\quad$ V_2

$\vec{V}_1$ $\quad$ $\vec{V}_2$

EXAMPLE 8.1.6. Compute $\mathbf{V}_1$ and $\mathbf{V}_2$ in the circuit below using nodal analysis.



$\vec{V}_s = 10\angle 45^\circ$ V

$\vec{Z}_1 = 4\ \Omega$

$\vec{Z}_2 = -3j\ \Omega$

$\vec{Z}_3 = 6j\ \Omega$

$\vec{Z}_4 = 12\ \Omega$

$\vec{I}_s = 3\angle 0^\circ$ A

@ supernode

$$-\vec{I}_s + \frac{\vec{V}_1}{\vec{Z}_2} + \frac{\vec{V}_2}{\vec{Z}_3} + \frac{\vec{V}_2}{\vec{Z}_4} + \cancel{\frac{\vec{V}_1 - \vec{V}_2}{\vec{Z}_1}} + \cancel{\frac{\vec{V}_2 - \vec{V}_1}{\vec{Z}_1}} = 0$$

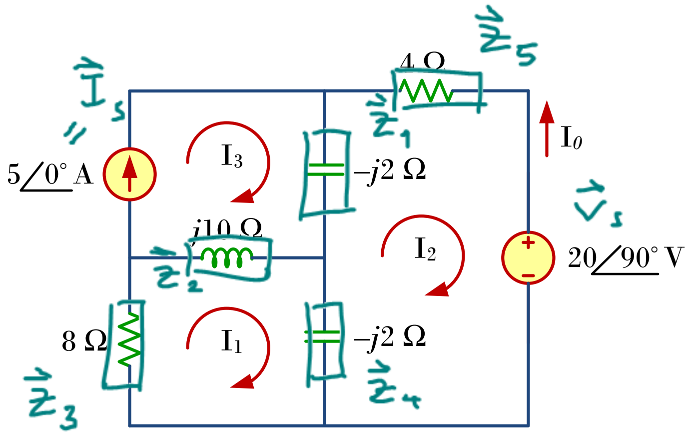
Supernode: $\vec{V}_s = \vec{V}_1 - \vec{V}_2 \Rightarrow \vec{V}_1 = \vec{V}_s + \vec{V}_2$

$$\frac{\vec{V}_s + \vec{V}_2}{\vec{Z}_2} + \vec{V}_2\left(\frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}\right) = \vec{I}_s$$

$$\vec{V}_2 = \frac{\vec{I}_s - \frac{\vec{V}_s}{\vec{Z}_2}}{\frac{1}{\vec{Z}_2} + \frac{1}{\vec{Z}_3} + \frac{1}{\vec{Z}_4}} = 31.4\angle -87^\circ$$

$$\vec{V}_1 = \vec{V}_s + \vec{V}_2 = 25.8\angle -70.5^\circ$$

EXAMPLE 8.1.7. Determine current $\mathbf{I}_o$ in the circuit below using mesh analysis.



$\vec{V}_s = 20\angle 90^\circ = 20j$

$\vec{I}_s = 5\angle 0^\circ = 5$

$\vec{Z}_1 = -2j\ \Omega$

$\vec{Z}_2 = 10j\ \Omega$

$\vec{Z}_3 = 8\ \Omega$

$\vec{Z}_4 = -2j\ \Omega$

$\vec{Z}_5 = 4\ \Omega$

$\vec{I}_3 = \vec{I}_s \Leftarrow$ mesh 3

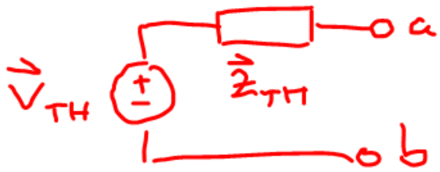
mesh 1: $-\vec{I}_1\vec{Z}_3 - (\vec{I}_1 - \vec{I}_3)\vec{Z}_2 - (\vec{I}_1 - \vec{I}_2)\vec{Z}_4 = 0$

mesh 2: $-(\vec{I}_2 - \vec{I}_1)\vec{Z}_4 - (\vec{I}_2 - \vec{I}_3)\vec{Z}_1 - \vec{I}_2\vec{Z}_5 - \vec{V}_s = 0$

$\Rightarrow \vec{I}_2 = 5 - 3.53j \Rightarrow \vec{I}_o = -\vec{I}_2 = -5 + 3.53j$

calc.

$= 6.12\angle 144.8^\circ$ A

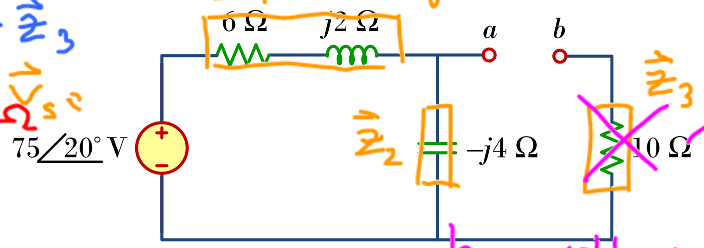


EXAMPLE 8.1.8. Find the Thevenin equivalent at terminals *a*-*b* of the circuit below.

$\vec{Z}_1 = 6 + 2j$

$\vec{Z}_{Th} = (\vec{Z}_1 \,//\, \vec{Z}_2) + \vec{Z}_3$

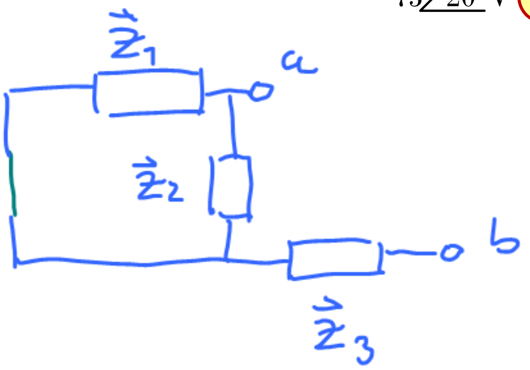
$= 12.4 - 3.2j \ \Omega$



$\vec{V}_{TH}$

Hanging branch no current ⇒ no voltage drop

voltage divider



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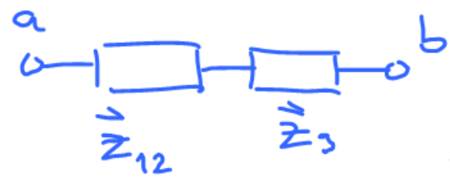


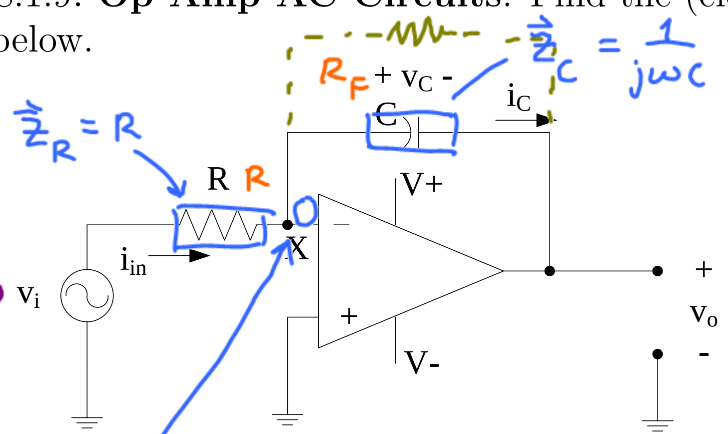
$$\vec{V}_{ab} = \frac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2}\vec{V}_s$$

$$\vec{V}_{ab} = \vec{V}_{TH} = 47.4\angle -51.6^\circ \text{ V}$$

EXAMPLE 8.1.9. **Op Amp AC Circuits**: Find the (closed-loop) gain of the circuit below.

$\vec{Z}_C = \frac{1}{j\omega C}$

$\vec{Z}_R = R$



Recall Inverting Amp.,

$$V_o = -\frac{R_F}{R} V_i$$

9V

$\omega = 0$

$f = 0$

Apply KCL @ "−"

$$\frac{0 - \vec{V}_i}{\vec{Z}_R} + \frac{0 - \vec{V}_o}{\vec{Z}_C} = 0$$

$$v_i(t) = V_m \cos(\omega t + \phi)$$

$$\vec{V}_o = -\frac{\vec{Z}_C}{\vec{Z}_R}\vec{V}_i$$

$$v_o(t) = \frac{V_m}{\omega RC}\cos(\omega t + \phi + 90^\circ)$$

$$= -\frac{1}{j\omega CR}\vec{V}_i$$

"Low pass filter"

$$= \frac{j}{\omega CR}\vec{V}_i$$

$1\angle 90^\circ$

8.2. Circuit With Multiple Sources Operating At Different Frequencies

A special care is needed if the circuit has multiple sources operating at different frequencies. In which case, one must add the responses due to the individual frequencies in the time domain. In other words, the superposition still works but

use super-position

(a) We must have a different frequency-domain circuit for each frequency. final ans
(b) The total response must be obtained by adding the individual response in the time domain.

8.2.1. Since the impedance depend on frequency, it is incorrect to try to add the responses in the phasor or frequency domain. To see this note that the exponential factor $e^{j\omega t}$ is implicit in sinusoidal analysis, and that factor would change for every angular frequency ω. In particular, although

$$\sum_i V_{mi}\cos(\omega t+\phi_i)=\sum_i \mathrm{Re}\left\{\mathbf{V}_i e^{j\omega t}\right\}=\mathrm{Re}\left\{\left(\sum_i \mathbf{V}_i\right)e^{j\omega t}\right\},$$

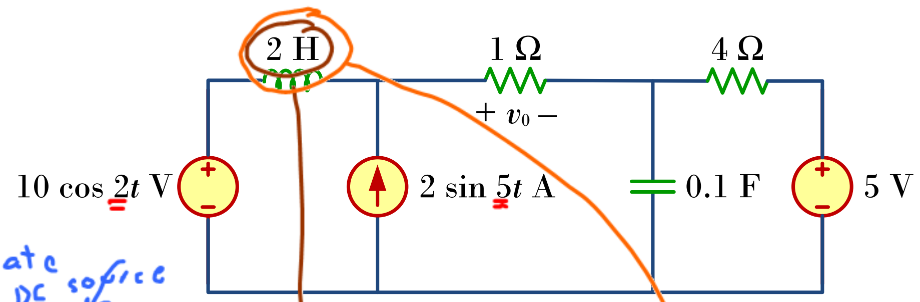
when we allow ω to be different for each sinusoid, generally

$$\sum_i V_{mi}\cos(\omega_i t+\phi_i)=\sum_i \mathrm{Re}\left\{\mathbf{V}_i e^{j\omega_i t}\right\}\neq\mathrm{Re}\left\{\left(\sum_i \mathbf{V}_i\right)e^{j\omega_i t}\right\}.$$

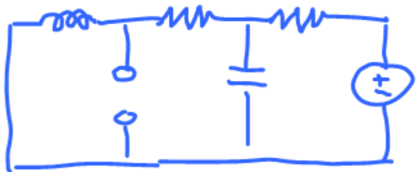
Therefore, it does not make sense to add responses at different frequencies in the phasor domain.

8.2.2. The Thevenin or Norton equivalent circuit (if needed) must be determined at each frequency and we have one equivalent circuit for each frequency.

Example 8.2.3. Find v_o in the circuit below using the superposition theorem.



activate only DC source

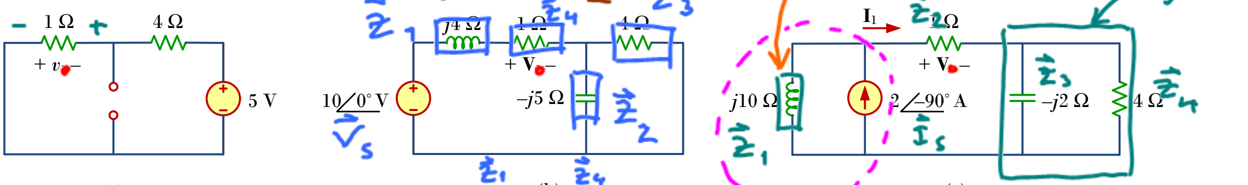


dc conditions

$j\omega L = j \times 2 \times 2 = 4j$

$j\omega L = j \times 5 \times 2 = 10j$

$\omega = 2$ $\omega = 5$ $\vec{Z}_3 // \vec{Z}_4$

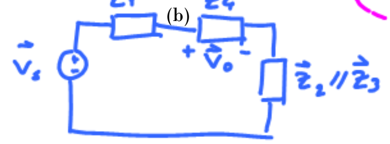


(a) (b) (c)

"$\vec{Z}_L = j\omega L = 0$

$\vec{Z}_C = \dfrac{1}{j\omega C} \to \infty$"

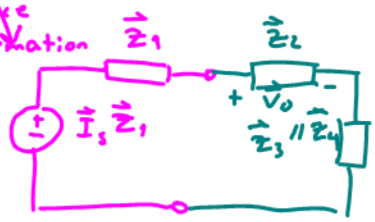
$v_o = -1$ V



$\vec{V}_o = \dfrac{\vec{Z}_4}{\vec{Z}_1 + \vec{Z}_4 + \vec{Z}_2 // \vec{Z}_3} \times \vec{V}_s = 2.5 \angle -30.78^\circ$

$v_o(t) = 2.5 \cos(2t - 30.78^\circ)$ V

source transformation



$\vec{V}_o = \vec{I}_s \vec{Z}_1 \times \dfrac{\vec{Z}_2}{\vec{Z}_1 + \vec{Z}_2 + \vec{Z}_3 // \vec{Z}_4}$

$= 2.33 \angle -77.9^\circ$ V

$v_o(t) = 2.33 \cos(5t - 77.9^\circ)$

$v_o(t) = -1 + 2.5 \cos(2t - 30.78^\circ) + 2.33 \cos(5t - 77.9^\circ)$ V

Using superposition:

If the frequencies of the subcircuits are different, transform to sinusoids first, then add your results in time domain to get the final answer.

If the frequencies of all subcircuits are the same, we may add the results in phasor forms first, then transform to sinusoid in time domain.

(sinusoids of the same frequency can be combined; it is easy to combine them as phasors.)